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Scattering of monopoles and decay of anti-monopolium

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Abstract

Undertaking the study of behaviour of a spin-1/2 non-Abelian magnetic monopole moving in the field of another non-Abelian monopole by solving Dirac's equation for energy eigen values and the Hamiltonian of this system has been shown to involve spin contribution. Spin momentum of a non-Abelian monopole has been shown to behave as extra energy source for non-Abelian monopole interacting with the field of another non-Abelian monopole. Interaction of spin and orbital moments of a non-Abelian monopole moving in the electromagnetic field has been analysed. S-matrix expansion, scattering amplitude and total Hamiltonian for monopole-anti-monopole scattering in non-Abelian gauge theory have also been undertaken. Study of bound state of a monopole and an anti-monopole in non-Abelian gauge theory has been carried out and it has been shown that this state is very short lived and decays in to two or three photons depending on the spin-statistics of the non-Abelian monopoles involved.

1. INTRODUCTION

Physicists were fascinated by magnetic monopole since its ingenious idea was put forward by Dirac(1) showing that mere existence of magnetic charge implies the quantization of electric charge. Ever since Dirac wrote down the quantization condition for magnetic charge, there have been many difficulties(1, 2) encountered in the scattering of magnetic monopole. The theory of magnetic monopole since its inception went through, two significant advancements, the quantum field theory of interacting electric and magnetic charges as developed by Schwinger(3) and the local Lagrangian quantum field theoretic formulation developed by Zwanziger(4) . In his approach Schwinger postulated both the Hamiltonian and the commutation rules, which while successful,

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seems an adhoc method of formulating a quantum field theory. The basic variables in his theory were the two transverse potentials $\mathbf{A}^T$and $\mathbf{B}^T$ which were not canonically conjugate. Zwanziger's formulation begins with a Lagrangian, but the price he pays for locality is the doubling of the number of Langrangian photon variables. As such the canonical and physical structure of his theory requires a great deal of clarification. Following the formulation of quantum field theory of magnetic charge closely patterned after Dirac(1) and Blagojevic et al.(5) , we have undertaken the study of Compton scattering of a photon with abelian monopole(6,7)and non-Abelian monopole(8) and it has been shown that the photon responsible for Compton scattering is highly energetic. We have also undertaken the study of S-matrix expansion and deduced Feynman diagrams for different charge particle and photon interactions in abelian(6) as well as in non-Abelian gauge theory(8). We have obtained(9), S-matrix expansion for monopole-anti-monopole scattering and bound state solutions of a monopole and an anti-monopole in abelian gauge theory and shown that this state is very short lived. Extending this work in the present paper, we have undertaken the study of Pauli's equation for non-Abelian monopoles. Analyzing Dirac's equation, the problem of interaction of spin and orbital angular momentum of this system has been investigated and the expression for Hamiltonian has been derived. We have also undertaken the study of S-matrix expansion for monopole-anti-monopole scattering in non-Abelian gauge theory. We have also obtained bound state solutions of a monopole and an anti-monopole in non-Abelian gauge theory.

2. NON-ABELIAN MONOPOLE IN ELECTROMAGNETIC FIELD OF ANOTHER NON-ABELIAN MONOPOLE

Dirac's equation for non-Abelina monopole may be written as

$$i\hbar \frac{\partial \psi}{\partial t} = c\hat{\alpha} \cdot \vec{p} + \hat{\beta} mc^2. \qquad ...(1)$$

By using the following transformation for momentum and energy operators in the above equation

$$p_\mu \to p_\mu - \left(1/c\right) g T^a B_\mu^a \qquad ...(2)$$

we get, Dirac's equation for a non-Abelian magnetic monopole moving in an electromagnetic field of another non-Abelian magnetic monopole;

$$i\hbar \frac{\partial \psi}{\partial t} = \left[c\hat{\alpha} \cdot \left\{ \vec{p} - \frac{1}{c} g T^a \vec{B}_i^a \right\} + \hat{\beta} m_o c^2 + g T^a B_o^a \right] \psi. \qquad ...(3)$$

Here g is the magnetic charge of non-Abelian monopole and B_μ^a is the four potential of electromagnetic field of non-Abelian monopole. The generators of gauge transformations in internal charge space may be considered as T^a with a=1, 2, 3 for SO(3) internal symmetry where matrices T^a are given by

$$\left(T^{c}\right)_{ab} = -i \in_{abc} \qquad \text{...(4)}$$

where $\in_{abc}$ is the usual Levi-Civita symbol. These matrices satisfy the following commutation rule;

$$\left[T^{a}, T^{b}\right] = i \in_{abc} T^{c}. \qquad \text{...(5)}$$

The relativistic energy of the particle includes also its rest energy $m_o c^2$. This must be excluded in arriving at the non-relativistic approximation, and we therefore replace ψ by a function ψ' defined as follows;

$$\psi = \begin{pmatrix} \psi^{1/2}_{1,1/2} \\ \psi^{1/2}_{1,-1/2} \end{pmatrix} = \begin{pmatrix} \phi \\ \chi \end{pmatrix} = \psi' e^{-im_o c^2 t/\hbar} \qquad \text{...(6)}$$

where ϕ and χ are two- component functions.

Then

$$\left(i\hbar \frac{\partial}{\partial t} + m_o c^2\right)\psi' = \left\{c\hat{\alpha} \cdot \left(\vec{p} - \frac{g}{c} T^a \vec{B}^a_i\right) + \hat{\beta} m_0 c^2 + g T^a \vec{B}^a_o\right\}\psi' \cdot \qquad \text{...(7)}$$

Equation (7) yields the following equations;

$$\left(i\hbar \frac{\partial}{\partial t} - g T^a B^a_o\right)\phi' = c\hat{\sigma} \cdot \left(\vec{p} - \frac{g}{c} T^a \vec{B}^a_i\right)\chi' \qquad \text{...(8)}$$

$$\left(i\hbar \frac{\partial}{\partial t} - g T^a B^a_o + 2 m_o c^2\right)\chi' = c\hat{\sigma} \cdot \left(\vec{p} - \frac{g}{c} T^a \vec{B}^a_i\right)\phi' \qquad \text{...(9)}$$

In the first approximation, the term $2m_o c^2 \chi$ is dominant and retained on the left hand side of (9), which gives

$$\chi' = \frac{1}{2m_o c} \hat{\sigma} \cdot \left(\vec{p} - \frac{g}{c} T^a \vec{B}^a_i\right)\phi'. \qquad \text{...(10)}$$

Substituting this in eqn. (8), we get

$$i\hbar \frac{\partial \phi'}{\partial t} = \left[\frac{1}{2m_o}\left\{\vec{p} - \frac{g}{c} T^a \vec{B}^a_i\right\}^2 + g T^a \vec{B}^a_o - \frac{g\hbar}{2m_o c}\left(\hat{\sigma} \cdot \vec{k}\right)\right]\phi' = \hat{H}\phi' \qquad \text{...(11)}$$

where

$$\vec{k} = \vec{\nabla} \times T^a \vec{B}^a_i \cdot \qquad \text{...(12)}$$

Eqn.(11) is analogous to Pauli's equation for an electron moving in electromagnetic field. It has the following extra spin contribution in the energy gained by non-Abelian monopole while moving in electromagnetic field of another monopole

$$E' = \frac{-g\hbar}{2m_o c}\left(\hat{\sigma} \cdot \vec{k}\right). \qquad \text{...(13)}$$

This equation can also be written as

$$E' = -\mu_g \cdot \vec{k} = -\mu_{g'}\left(\hat{\sigma} \cdot \vec{k}\right) \qquad ...(14)$$

where

$$\mu_{g'} = \frac{g\hbar}{2m_o c} \qquad ...(15)$$

describes Bohr magneton for the system of two non-Abelian monopoles and

$$\mu_g = \mu_{g'}\hat{\sigma} \qquad ...(16)$$

is defined as spin momentum of non-Abelian monopole in the field of another monopole. Consequently, extra-energy term in the Hamiltonian, may be interpreted as the energy of interaction of the spin moment of a non-Abelain monopole with a vector field, resulting from the space rotation of four potential B^a_μ. The third component of the spin moment operator for magnetic monopole may be written as:

$$\left(\mu_g\right)_3 = \frac{g\hbar}{2m_o c}\hat{\sigma}_3 \qquad ...(17)$$

the eigen values of which are

$$\pm\frac{g\hbar}{2m_o c} = \pm\mu_{g'}. \qquad ...(18)$$

For deriving the wave equation corresponding to Schrodinger's equation, in the second approximation $\vec{B}^a_i = o$, we must replace ϕ by another (two-component) function ϕ_{Sch}, for which the time independent integral would be of the form $\int\left|\phi_{Sch}\right|^2 d^3x$, as it should be for Schrodinger's equation.

To obtain the required transformation, we write the condition

$$\int \phi^*_{Sch}\phi_{Sch}d^3x = \int\left\{\phi^*\phi + \frac{\hbar^2}{4m_o^2c^2}\left(\vec{\nabla}\phi^* \cdot \hat{\sigma}\right)\left(\hat{\sigma} \cdot \vec{\nabla}\phi\right)\right\}d^3x$$

$$= \int\left\{\phi^*\phi - \frac{\hbar^2}{8m_o^2c^2}\left(\phi^*\Delta\phi + \phi\Delta\phi^*\right)\right\}d^3x \qquad ...(19)$$

thus

$$\phi_{Sch} = \left(1 + \frac{\vec{p}^2}{8m_o^2c^2}\right)\phi, \phi = \left(1 - \frac{\vec{p}^2}{8m_o^2c^2}\right)\phi_{Sch} \qquad ...(20)$$

To simplify the notation we shall assume a stationary state, replacing the operator $i\hbar\frac{\partial}{\partial t}$ by the energy ε . In the next approximation after (10) we have from (9) with $\vec{B}^a_i = 0$

$$\chi = \frac{1}{2m_o c}\left(1 - \frac{\varepsilon - gT^a B_o^a}{2m_o c^2}\right)(\hat{\sigma} \cdot \vec{p})\phi. \qquad ...(21)$$

This is to be substituted in (8) and ϕ then be replaced by ϕ_{Sch} according to (20), omitting all terms of higher order than $1/c^2$. We get

$$\varepsilon\phi_{Sch} = \left[\frac{\vec{p}^2}{2m_o} + gT^a B_0 - \frac{\vec{p}^4}{8m_o^3 c^2} + \frac{g}{4m_o^2 c^2}\left\{(\hat{\sigma} \cdot \vec{p})T^a B_o^a(\hat{\sigma} \cdot \vec{p}) - \frac{1}{2}\left(\vec{p}^2 T^a B_o^a + T^a B_o^a \vec{p}^2\right)\right\}\right]\phi_{Sch} = \hat{H}\phi_{Sch} \qquad ...(22)$$

The final expression for the Hamiltonian can be written as

$$\hat{H} = \frac{\vec{p}^2}{2m_0} + gT^a B_o^a - \frac{\vec{p}^4}{8m_o^3 c^2} + \frac{g\hbar}{4m_o^2 c^2}\hat{\sigma} \cdot \vec{\nabla} T^a B_o^a \times \vec{p} + \frac{g\hbar^2}{8m_o^2 c^2} \operatorname{div} \vec{\nabla} T^a B_o^a \qquad ...(23)$$

The last three terms are required corrections of order $1/c^2$. The first of these three terms is due to the relativistic dependence of the kinetic energy on the momentum. The second, which may be called the spin-orbit interaction energy, is the energy of the interaction of the moment with the field associated with non-Abelian monopoles. The last term may be interpreted as the contact interaction operator for non-Abelian monopoles, which is analogous to the term introduced by Darwin(10) for the electronic case. This term vanishes except at the locations of magnetic charge, creating external field. In a spherically symmetric magnetic field

$$\vec{\nabla} B_o^a = \frac{\vec{r}}{r}\frac{\partial B_o^a}{\partial r} \qquad ...(24)$$

the spin-orbit interaction operator can be put in the form:

$$\frac{g\hbar T^a}{2m_o c^2 r}\frac{\partial B_o^a}{\partial r}\hat{S} \cdot \hat{l} \qquad ...(25)$$

where $\hat{S} = \frac{1}{2}\hat{\sigma}$ and $\hat{l} = \left[\vec{r} \times \vec{p} - \left(\frac{\vec{r} \cdot \hat{r}}{r}\right)\vec{T}\right]$ are spin and orbital angular momentum operators for non-Abelian monopoles. This expression clearly demonstrates that besides the contribution of magnetic field, the interaction of spin and orbital angular momenta of moving non-Abelian monopole also contributes to the energy operator.

3. S-MATRIX EXPANSION FOR NON-ABELIAN MONOPOLES

In this section we shall undertake the monopole-anti-monopole scattering in non-Abelian gauge theory with the help of S-matrix expansion technique so that the results are generalization of Abelian gauge theory(9). The complete gauge invariant Lagrangian density for the interaction of non-Abelian monopole with the field of another monopole may be written as

$$L = -\left(\frac{1}{4}\right)H^{a}_{\mu\nu}H^{a\mu\nu} + \overline{\psi}\, i\,\gamma_{\mu}D_{\mu}\psi - m_{o}\overline{\psi}\psi \qquad ...(26)$$

where $H^{a}_{\mu\nu}$ is the electromagnetic field strength tensor of non-Abelian monopole, given as

$$H^{a}_{\mu\nu} = \partial_{\mu}B^{a}_{\nu} - \partial_{\nu}B^{a}_{\mu} + g \in^{abc} B^{b}_{\mu}B^{c}_{\nu} \qquad ...(27)$$

B^{a}_{μ} in equation(27) is the magnetic four-potential of non-Abelian monopole, g is the magnetic charge of non-Abelian monopole and the covariant derivative is

$$D_{\mu} = \partial_{\mu} - igT^{a}B^{a}_{\mu}\,. \qquad ...(28)$$

The generators of gauge transformations in internal charge space may be considered as T^{a} with $a = 1,2,3$ for SO(3) internal symmetry where matrices T^{a} are given by

$$\left(T^{c}\right)_{ab} = -i \in_{abc} \qquad ...(29)$$

where $\in_{abc}$ is the usual Levi-Civita symbol. These matrices satisfy the following commutation rule;

$$\left[T^{a}, T^{b}\right] = i \in_{abc} T^{c}\,. \qquad ...(30)$$

Using minimal replacement of equation (28) the Lagrangian density takes the following form

$$\hat{L}(x) = -\left(\frac{1}{4}\right)H^{a}_{\mu\nu}(x)H^{a\mu\nu}(x) + i\overline{\psi}(x)\hat{\gamma}_{\mu}\left(\partial_{\mu} - igT^{a}\hat{B}^{a}_{\mu}\right)\hat{\psi}(x) - m_{o}\hat{\overline{\psi}}(x)\hat{\psi}(x) \qquad ...(31)$$

which includes the interaction Lagrangian density

$$\hat{L}_{I}(x) = g\hat{\overline{\psi}}(x)\hat{\gamma}_{\mu}\hat{B}^{a}_{\mu}V^{a}_{\mu}T^{a}\hat{\psi}(x)\,. \qquad ...(32)$$

The interaction Hamiltonian density may be deduced in the form

$$\hat{H}_{I}(x) = -g\hat{\overline{\psi}}(x)\hat{\gamma}_{\mu}\hat{B}^{a}_{\mu}T^{a}\hat{\psi}(x) = -gN\left(\hat{\overline{\psi}}\hat{\gamma}_{\mu}\hat{B}^{a}_{\mu}T^{a}\hat{\psi}_{g}\right)_{x}\,. \qquad ...(33)$$

The perturbative series solutions (i.e.S-matrix expansion) for the interaction of non-Abelian monopole with the field of another monopole, may be obtained by writing the S-matrix expansion, with perturbative Hamiltonian $\hat{H}_{I}(t)$ in the interaction picture, defined as

$$\hat{S} = \sum_{n=o}^{\infty}\frac{(-i)^{n}}{n!}\int_{-\infty}^{\infty}dt_{1}\int_{-\infty}^{\infty}dt_{2}\ldots\int_{-\infty}^{\infty}dt_{n}P\left[\hat{H}_{I}(t_{1})\hat{H}_{I}(t_{2})\ldots\hat{H}_{I}(t_{n})\right] \qquad ...(34)$$

where P is the Dyson chronological product and $\hat{H}_{I}$ (t) is given as

$$\hat{H}_{I}(t) = \int d\vec{x}\hat{H}_{I}(x) \qquad ...(35)$$

$\hat{H}_I(x)$ of this equation is given by equation (33). Substituting the value of $\hat{H}_I(t)$ from equation (35) in to equation (34), we get

$$\hat{S} = \sum_{n=o}^{\infty} \frac{(-i)^n}{n!} \int d^4x_1 \int d^4x_2 \ldots\ldots \int d^4x_n T\left[\hat{H}_I(x_1)\hat{H}_I(x_2)\ldots\ldots \hat{H}_I(x_n)\right] \quad ...(36)$$

where T denotes the Wick's chronological product. With the help of equations (33) and (36), we get

$$\hat{S} = \sum_{n=o}^{\infty} \frac{(i)^n}{n!} g^n \int d^4x_1 \int d^4x_2 \ldots \int d^4x_n T'\left[\left(\hat{\bar{\psi}}\hat{\gamma}_\mu \hat{B}^a_\mu T^a \hat{\psi}\right)_{x_1} \left(\hat{\bar{\psi}}\hat{\gamma}_\upsilon \hat{B}^b_\upsilon T^b \hat{\psi}\right)_{x_2} \cdots \right.$$

$$\left. \ldots \left(\hat{\bar{\psi}}\hat{\gamma}_\rho \hat{B}^c_\rho T^c \hat{\psi}\right)_{x_n}\right]. \quad ...(37)$$

For studying monopole-anti-monopole scattering in non-Abelian gauge theory[8] we obtained the following equation for the third part of second order S-matrix from equation (37):

$$\left[S_3^{(2)}\right]_{fi} = -\frac{ig^2}{2!} \int d^4x_1 \int d^4x_2 \hat{\bar{\psi}}^-(x_1)\hat{\gamma}_\mu T^a \hat{\psi}^-(x_2)\hat{\psi}^+(x_1)\hat{\bar{\psi}}^+(x_2)\hat{\gamma}_\upsilon T^b \left\{D_F^{ab}(x_1 - x_2)\right\}_{\mu\upsilon}$$

$$= -\frac{ig^2}{2!} \int d^4x_1 \int d^4x_2 \left[\sqrt{\frac{m_o}{VE_{p'_-}}}\,\bar{u}(p'_-)e^{-ip'_-x_1}\right]\hat{\gamma}_\mu T^a \left[\sqrt{\frac{m_o}{VE_{p'_+}}}\,u(-p'_+)e^{-ip'_+x_2}\right]$$

$$\delta_{ab}\int \frac{d^4k}{(2\pi)^4} e^{ik(x_1 - x_2)}\left[-g^{\mu\nu} + (1-\xi)\frac{k^\mu k^\nu}{k^2}\right]\frac{1}{k^2 + i\in}\gamma_\nu T^b$$

$$\left[\sqrt{\frac{m_o}{VE_{p_-}}}\,u(p_-)e^{ip_-x_1}\right]\left[\sqrt{\frac{m_o}{VE_{p_+}}}\,\bar{u}(-p_+)e^{ip_+x_2}\right]. \quad ...(38)$$

where ξ is some orbitrary constant coefficient, the gauge parameter $\xi=1$ for Feyman-'t Hooft gauge and $\xi = 0$ for Landau gauge.

Determining the integration with respect to x_1 and x_2, we get

$$\int d^4x_1 e^{ix_1(k-p'_- + p_-)} \int d^4x_2 e^{ix_2(-k-p'_+ + p_+)}$$

$$= (2\pi)^4\delta^4(k - p'_- + p_-)(2\pi)^4\delta^4(-k - p'_+ + p_+) \quad ...(39)$$

Substituting relation (39) into equation (38), we get

$$\left[S_3^{(2)}\right]_{fi} = i(2\pi)^4\delta^4(p_+ + p_- - p'_+ - p'_-)$$

$$\sqrt{\left(\frac{m_o}{VE_{p'_-}}\right)\left(\frac{m_o}{VE_{p'_+}}\right)\left(\frac{m_o}{VE_{p_-}}\right)\left(\frac{m_o}{VE_{p_+}}\right)}\,M_{fi} \quad ...(40)$$

where Feynman amplitude is given by

$$M_{fi} = -g^2\left[\bar{u}(p'_-)\hat{\gamma}_\mu T^a u(p_-)\right]\left\{D_F^{ab}(k)\right\}_{\mu\upsilon}\left[\bar{u}(-p_+)\hat{\gamma}_\upsilon T^b u(-p'_+)\right] \qquad ...(41)$$

which may be written as follows for monopole-anti-monopole scattering in non-Abelian gauge theory:

$$M_{fi} = -g^2\left[\bar{u}(p'_-)\hat{\gamma}_\mu T^a u(p_-)\right] D_{\mu\upsilon}^{ab}(k)\left[\bar{u}(-p_+)\hat{\gamma}_\upsilon T^b u(-p'_+)\right]$$
$$+g^2\left[\bar{u}(-p_+)\hat{\gamma}_\mu T^a u(p_-)\right] D_{\mu\upsilon}^{ab}(k)\left[\bar{u}(p'_-)\hat{\gamma}_\upsilon T^b u(-p'_+)\right]. \qquad ...(42)$$

We know,

$$T^a = T^b = \frac{\tau}{2} \text{ and } \left\{\tau_i, \tau_j\right\} = 2\delta_{ij} \Rightarrow \tau^2 = 1 \qquad ...(43)$$

where τ_i and τ_j are Pauli matrices.

Thus from equations (43) and (42), we get

$$M_{fi} = -\frac{g^2}{4}\left[\bar{u}(p'_-)\hat{\gamma}_\mu u(p_-)\right] D_{\mu\upsilon}(k)\left[\bar{u}(-p_+)\hat{\gamma}_\upsilon u(-p'_+)\right]$$
$$+\frac{g^2}{4}\left[\bar{u}(-p_+)\hat{\gamma}_\mu u(p_-)\right] D_{\mu\upsilon}(k)\left[\bar{u}(p'_-)\hat{\gamma}_\upsilon u(-p'_+)\right]. \qquad ...(44)$$

The first term of this equation describes the scattering process and second term leads to the annihilation process. Let us consider in the first term of eqn. (44) that the two particles are same, with mass m_0. The photon propagator $D_{\mu\upsilon}(k)$ is chosen in the Coulomb gauge:

$$D_{oo} = -\frac{4\pi}{k^2}, \; D_{oi} = o, \; D_{ik} = \frac{4\pi}{k^2 - \omega^2/c^2}\left(\delta_{ik} - \frac{k_i k_k}{\vec{k}^2}\right). \qquad ...(45)$$

Then the first term of scattering amplitude (44) is

$$M_{fi}^{(scatt)} = -\frac{g^2}{4}\left[(\bar{u}'_-\hat{\gamma}_o u_-)(\bar{u}_+\hat{\gamma}_o u'_+)D_{oo} + (\bar{u}'_-\hat{\gamma}_i u_-)(\bar{u}_+\hat{\gamma}_k u'_+)D_{ik}\right]. \qquad ...(46)$$

Neglecting the terms containing 1/c, the second term in the braces vanishes, and the first term gives

$$M_{fi}^{(scatt)} = -4m_o^2\left(w_-'^{(o)^*} w_-^{(o)}\right)\left(w_+^{(o)^*} w_+'^{(o)}\right)\cup(\vec{k}) \qquad ...(47)$$

where

$$\cup(\vec{k}) = -\pi g^2/\vec{k}^2, \qquad ...(48)$$

and $w_-^{(o)}, w_+^{(o)}$ denote the two components spinor amplitudes of the non-relativistic plane waves.

The Schrodinger wave function of the free particle ϕ_{Sch} in the next approximation with respect to 1/c satisfies the equation

$$\hat{H}^{(o)} = \left(\varepsilon - m_0c^2\right)\phi_{Sch},$$

$$\hat{H}^{(o)} = \frac{\vec{p}^2}{2m_o} - \frac{\vec{p}^4}{8m_o^3c^2}, \vec{p} = -i\hbar\vec{\nabla} \qquad ...(49)$$

which includes the next term in the expansion of the relativistic expression for the kinetic energy. The spinor amplitude of this plane wave will be denoted w , which tends to $w^{(o)}$ as $1/c \rightarrow o$.

With the help of formula (20), the bispinor amplitude u of the free particle can be written in terms of Schrodinger amplitude w, as

$$u = \sqrt{(2m_o)}\begin{pmatrix}\left(1 - \vec{p}^2/8m_o^2c^2\right)w \\ \left(\hat{\sigma}\cdot\vec{p}/2m_oc\right)w\end{pmatrix}. \qquad ...(50)$$

Which gives

$$\bar{u}'_-\hat{\gamma}_o u_- = u'^*_- u_- = 2m_0\left(1 - \frac{\vec{p}'^2_- + \vec{p}^2_-}{8m_0^2c^2}\right)w'^*_- w_-$$

$$+ \frac{1}{2m_oc^2} w'^*_-\left(\hat{\sigma}\cdot\vec{p}'_-\right)\left(\hat{\sigma}\cdot\hat{p}'_-\right)w_-$$

$$= 2m_o w'^*_-\left[1 - \frac{\vec{k}^2}{8m_o^2c^2} + \frac{i\hat{\sigma}\cdot\vec{k}\times\vec{p}_-}{4m_0^2c^2}\right]w_-,$$

$$\bar{u}'_-\hat{\gamma}u_- = u'^*_-\hat{\alpha}u_-$$

$$= \left(1/c\right)w'^*_-\left[\hat{\sigma}\left(\hat{\sigma}\cdot\vec{p}_-\right) + \left(\hat{\sigma}\cdot\vec{p}'_-\right)\hat{\sigma}\right]w_-$$

$$= \left(1/c\right)w'^*_-\left[i\hat{\sigma}\times\vec{k} + 2\vec{p}_- + \vec{k}\right]w_-,$$

where $\vec{k} = \vec{p}'_- - \vec{p}_- = \vec{p}_+ - \vec{p}'_+$.

The corresponding expressions for $\left(\bar{u}_+\hat{\gamma}_o u'_+\right)$ and $\left(\bar{u}_+\hat{\gamma}u'_+\right)$ differ in that the suffix - is replaced by + and $\vec{k}$ by $-\vec{k}$. We now substitute these expressions in eqn. (2.177). Since the product $\left(\bar{u}'_-\hat{\gamma}u_-\right)\left(\bar{u}_+\hat{\gamma}u'_+\right)$ already contains the factor $1/c^2$, the term ω^2/c^2 in the denominator of D_{ik} may be neglected. The $M_{fi}^{(scatt)}$ is then

$$M_{fi}^{(scatt)} = -4m_o^2\left[w'^*_- w^*_+ \cup\left(\vec{p}_-, \vec{p}_+, \vec{k}\right)w_- w'_+\right] \qquad ...(51)$$

where

$$\bigcup\left(\vec{p}_-, \vec{p}_+, \vec{k}\right) = -\pi g^2\left[\frac{1}{\vec{k}^2} - \frac{1}{4m_o^2c^2} + \frac{\left(\vec{k}\cdot\vec{p}_-\right)\left(\vec{k}\cdot\vec{p}_+\right)}{m_o^2\mathbf{k}^4} - \frac{\vec{p}_-\cdot\vec{p}_+}{m_o^2\vec{k}^2} + \frac{i\sigma_-\cdot\vec{k}\times\vec{p}_-}{4m_o^2c^2\vec{k}^2} - \frac{i\sigma_-\cdot\vec{k}\times\vec{p}_+}{2m_o^2c^2\vec{k}^2} + \frac{i\sigma_+\cdot\vec{k}\times\vec{p}_+}{4m_o^2c^2\vec{k}^2} + \frac{i\sigma_+\cdot\vec{k}\times\vec{p}_-}{2m_o^2c^2\vec{k}^2} + \frac{\left(\hat{\sigma}_-\cdot\vec{k}\right)\left(\hat{\sigma}_+\cdot\vec{k}\right)}{4m_o^2c^2\vec{k}^2} - \frac{\hat{\sigma}_-\cdot\hat{\sigma}_+}{4m_o^2c^2}\right]; \quad ...(52)$$

the suffix - and + to the Pauli matrices indicate the spinor indices on which they act, $\hat{\sigma}_-$ act on w_- and $\hat{\sigma}_+$ on w_+.

The function $\bigcup\left(\vec{p}_-, \vec{p}_+, \vec{k}\right)$ is the particle interaction operator in the momentum representation. We obtain

$$\bigcup\left(\vec{p}_-, \vec{p}_+, \vec{r}\right) = -\frac{g^2}{4r} + \frac{g^2\pi\hbar^2}{4m_o^2c^2}\delta(\vec{r}) + \frac{g^2}{8m_o^2c^2r}\left[\vec{p}_-\cdot\vec{p}_+ + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p}_-)\vec{p}_+}{r^2}\right] + \frac{g^2\hbar}{16m_o^2c^2r^3}\vec{r}\times\vec{p}_-\cdot\hat{\sigma}_- - \frac{g^2\hbar}{16m_o^2c^2r^3}\vec{r}\times\vec{p}_+\cdot\hat{\sigma}_+ + \frac{g^2\hbar}{8m_o^2c^2r^3}\left[\vec{r}\times\vec{p}_-\cdot\hat{\sigma}_+ - \vec{r}\times\vec{p}_+\cdot\hat{\sigma}_-\right] - \frac{g^2\hbar^2}{16m_o^2c^2}\left[\frac{\hat{\sigma}_-\cdot\hat{\sigma}_+}{r^3} - 3\frac{(\hat{\sigma}_-\cdot\vec{r})(\hat{\sigma}_+\cdot\vec{r})}{r^5} - \frac{8\pi}{3}\hat{\sigma}_-\cdot\hat{\sigma}_+\delta(\vec{r})\right] \quad ...(53)$$

in coordinate representation by the usual techniques.

The total Hamiltonian of the monopole-anti-monopole scattering in non-Abelian gauge theory is

$$\hat{H} = \hat{H}^{(0)} + \bigcup\left(\vec{p}_-, \vec{p}_+, \vec{r}\right) \quad ...(54)$$

where $\hat{H}^{(0)} = \hat{H}_-^{(0)} + \hat{H}_+^{(0)}$ is the free particle Hamiltonian, from eqn. (49);

$$\hat{H}^{(0)} = \frac{1}{2m_o}\left(\vec{p}_-^2 + \vec{p}_+^2\right) - \frac{1}{8m_o^3c^2}\left(\vec{p}_-^4 + \vec{p}_+^4\right). \quad ...(55)$$

Let us now study the transformation of the second term of the eqn. (44). Here the photon propagator is chosen in the ordinary gauge;

$$D_{\mu\nu} = \frac{4\pi}{k^2}g_{\mu\nu} = \frac{4\pi}{\frac{\omega^2}{c^2} - \vec{k}^2}g_{\mu\nu}.$$

with $k = p_+ + p_-$, and the particles are non-relativistic. Thus we get

$$\frac{\omega^2}{c^2} \equiv \frac{(\varepsilon_+ + \varepsilon_-)^2}{c^2} \approx 4m_o^2c^2 \rangle\rangle (\vec{p}_+ + \vec{p}_-)^2 \equiv \vec{k}^2. \qquad ...(56)$$

We can write photon propagtor as

$$D_{\mu\nu} \approx \left(\frac{\pi}{m_o^2c^2}\right) g_{\mu\nu}.$$

The amplitude u (p) in the zero-order approximation is given as ;

$$u(-p_-) = \sqrt{2m_o}\begin{pmatrix} w_-^{(o)} \\ 0 \end{pmatrix}, \quad u(-p_+) = \sqrt{2m_o}\begin{pmatrix} 0 \\ w^{(o)} \end{pmatrix},$$

where $w_-^{(o)}, w^{(o)}$ are the three dimensional spinors, the index (0) will hence-forward be neglected. For these amplitudes, we have

$$\bar{u}(-p_+)\hat{\gamma}_o u(p_-) = u^*(-p_+)u(p_-) = 0,$$

$$\bar{u}(-p_+)\hat{\gamma}\, u(p_-) = u^*(-p_+)\hat{\alpha}\, u(p_-) = 2m_o\left(w^*\hat{\sigma}w_-\right).$$

With the help of these expressions, the annihilation term in the scattering amplitude(44) becomes

$$M_{fi}^{(ann)} = -g^2 \frac{\pi}{4m_o^2c^2}(2m_0)^2\left(w^*\hat{\sigma}w_-\right)\left(w_-'^*\hat{\sigma}w'\right). \qquad ...(57)$$

The non-Abelian anti-monopole amplitudes are obtained from $u(-p_+)$ by charge conjugtion and the corresponding spinors (which are denoted by w_+) are related to w by

$$w_+ = \hat{\sigma}_y w^*.$$

Thus we get

$$w^* = \sigma_y w_+ = -w_+\sigma_y, w = -\sigma_y w_+^*. \qquad ...(58)$$

The scattering amplitude must be brought to a form in which the non-Abelian monopole spinors (w_- and w_-') are contracted and likewise the non-Abelian anti-monopole spinors (w_+ and w_+'). This is obtained by the formula

$$\left(w^*\hat{\sigma}w_-\right)\left(w_-'^*\hat{\sigma}w'\right) = \frac{1}{2}\left(w_-'^*w_-\right)\left(w^*w'\right) - \frac{1}{2}\left(w_-'^*\hat{\sigma}w_-\right)\left(w^*\hat{\sigma}w'\right), \qquad ...(59)$$

Finally, expressing w and w' in terms of w_+ and w_+' by (58), we get

$$\left(w^*w'\right) = \left(w_+'^*w_+\right)$$

$$\left(w^*\hat{\sigma}w'\right) = -\left(w_+'^*\hat{\sigma}w_+\right) \qquad ...(60)$$

Substituting (60) in (59) and then in (57), we find

$$M_{fi}^{(ann)} = -4m_0^2\left(w_-'^*w_+'^*\left[\frac{\pi g^2}{8m_o^2c^2}\left(3 + \hat{\sigma}_+ \cdot \hat{\sigma}_-\right)\right]w_-w_+\right).$$

The expression in the square brackets is the interaction operator in the momentum representation. The corresponding coordinate operator is

$$U^{(ann)}(\vec{r}) = \frac{\pi\hbar^2 g^2}{2m_0^2\, c^2}\left(3 + \hat{\sigma}_+ \cdot \hat{\sigma}_-\right)\delta(\vec{r}) \qquad ...(61)$$

where $\vec{r} = \vec{r}_- - \vec{r}_+$.

The total monopole-anti-monopole interaction operator in non-Abelian gauge theory is $\hat{U} + \hat{U}^{(ann)}$, with $\hat{U}$ and $\hat{U}^{(ann)}$ given by equations (53) and (61) respectively.

4. BOUND STATE OF A MONOPOLE AND AN ANTI-MONOPOLE IN NON-ABELIAN GAUGE THEORY

The results of the preceding section can be applied to bound state of a monopole and anti-monopole in non-Abelian gauge theory. In the centre-of-mass system, the non-Abelian monopole and non-Abelian anti-monopole momentum operators in non-Abelian anti-monopolium are $\vec{p}_- = -\vec{p}_+ \equiv \vec{p}$, where $\vec{p} = -i\hbar\vec{\nabla}$ is the operator of the momentum of relative motion corresponding to relative position vector $\vec{r} = \vec{r}_- - \vec{r}_+$. The total Hamiltonian for the system is

$$\hat{H} = \frac{\vec{p}^2}{m_o} - \frac{g^2}{4r} + \hat{V}_1 + \hat{V}_2 + \hat{V}_3$$

where

$$\hat{V}_1 = -\frac{\vec{p}^4}{4m_o^3c^2} + \frac{g^2\pi\hbar^2}{4m_o^2c^2}\delta(\vec{r}) - \frac{g^2}{8m_o^3c^2}\left[\vec{p}^2 + \frac{\vec{r}\cdot(\vec{r}\cdot\vec{p})\vec{p}}{r^2}\right]$$

$$\hat{V}_2 = \frac{3g^2\hbar^2}{8m_o^2c^2r^3}\,\hat{l}\cdot\hat{s}$$

$$\hat{V}_3 = \frac{3g^2\hbar^2}{8m_o^2c^2r^3}\left[\frac{(\hat{s}\cdot\vec{r})(\hat{s}\cdot\vec{r})}{r^2} - \frac{1}{3}\hat{s}^2\right] + \frac{g^2\pi\hbar^2}{4m_o^2c^2}\left(\frac{7}{3}\hat{s}^2 - 2\right)\delta(\vec{r}). \qquad ...(62)$$

Here $\hbar\hat{l} = \left[\vec{r}\times\vec{p} - \left(\frac{\vec{r}\cdot\hat{r}}{r}\right)\vec{T}\right]$ is the orbital angular momentum operator, $\hat{s} = \frac{1}{2}\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)$ the total spin operator of the system, square of which is $\hat{s}^2 = \frac{1}{2}\left(3 + \hat{\sigma}_+\cdot\hat{\sigma}_-\right)$. $\hat{V}_1$ includes all the purely orbital correction terms, $\hat{V}_2$ includes the spin-orbit interaction and $\hat{V}_3$ includes the spin-spin and annihilation interactions.

The unperturbed Hamiltonian is, given as

$$\hat{H}^{(o)} = \frac{\vec{p}^2}{m_o} - \frac{g^2}{4r} \quad . \qquad ...(63)$$

The energy levels of non-Abelian anti-monopolium therefore have absolute values;

$$E = -\frac{m_o g^4}{4\hbar^2 n^2} \qquad ...(64)$$

where n is the principal quantum number. The remaining terms in (62) cause a splitting of the levels of eqn. (64). The resulting levels are classified primarily by the values of the total angular momentum J. We also observe that particle spin operator appear in Hamiltonian (62) only through the sum $\hat{s}$. This implies that the Hamiltonian commutes with the squared total spin operator $\hat{s}^2$. The energy levels of non-Abelian anti-monopolium can therefore be classified by the total spin which takes the values s = 0 and s = 1. The levels with spin-0 are called non-Abelian para-anti-monopolium levels, and with spin-1 are non-Abelian ortho-anti-monopolium levels. The orbital magnetic moment of non-Abelian anti-monopolium is always zero, since for it $\vec{r}_+ \times \vec{p}_+ = \vec{r}_- \times \vec{p}_-$ and we have the operator

$$\hat{\mu}_l = \frac{g\hbar}{2m_o c}\left(\vec{r}_+ \times \vec{p}_+ - \vec{r}_- \times \vec{p}_-\right) = 0 \, . \qquad ...(65)$$

The spin magnetic moment operator,

$$\mu_s = \frac{g\hbar}{2m_o c}\left(\hat{\sigma}_+ - \hat{\sigma}_-\right) \qquad ...(66)$$

is not proportional to the total spin operator $\hat{s} = \frac{1}{2}\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)$, and the operator $\hat{s}^2$ and μ_s^2 do not commute. The splitting in energy (fine structure) is given by the mean value of correction terms in the Hamiltonian (62), calculated by means of the wave functions of the unperturbed states with different values of J = l [= 0,1, ...(n-1)]. When $\hat{s} = 0$, the only non- zero contributions come from $\hat{V}_1$ and the second term in $\hat{V}_3$. The energy levels of non-Abelian para-anti-monopolium can be written as

$$E_{nl} = -\frac{1}{4n^2} - \alpha_g^2 \frac{m_o g^4}{\hbar^2} \frac{1}{2n^3}\left(\frac{1}{2l+1} - \frac{11}{32n}\right) \qquad ...(67)$$

where $\alpha_g = g^2 / 4\pi \in_o \hbar c$ is the fine structure constant for the system.

Now we shall determine the lifetime of non-Abelian anti-monopolium.. The first process is two-photon decay from non-Abelian para-anti-monopolium and the second process is three-photon decay from non-Abelian ortho-anti-monopolium. The annihilation of a monopole and an anti- monopole (with four momenta p_- and $-p_+$) in non-Abelian gauge theory to form two-photons which are extra ordinarily energetic having momenta K_1 and K_2 as shown in fig. 1. For non-Abelian ortho-anti-monopolium annihilitaion form three-photon which are extra ordinarily energetic having momenta K_1 , K_2 and K_3 [as shown in fig 2].

Let $\bar{\sigma}_{2\gamma}$ be the cross section for two photons decay from a free pair averaged over the spin directions of both particles. In the non-relativistic limit

$$\bar{\sigma}_{2\gamma} = \frac{\pi g^4}{m_o^2 c^4} \frac{c}{v} \qquad ...(68)$$

where v is the relative velocity of the particles. The annihilation probability $\bar{w}_{2\gamma}$ is obtained on multiplying $\bar{\sigma}_{2\gamma}$ by the flux density $v|\psi(o)|^2$. The normalized wave function $\psi(r)$, of the non-Abelian anti-monopolium ground state is

$$\psi(r) = \frac{1}{\left(\pi a^3\right)^{1/2}} g^{-r/a} \quad ; \quad a = \frac{2\hbar^2}{m_o g^2} \qquad ...(69)$$

where a is the Bohr radius of non-Abelian anti-monopolium and m_o is the mass of non-Abelian magnetic monopole(11). The mean decay probability $\bar{w}_{2\gamma}$ is related to the non-Abelian para-anti-monopolium decay probability w_o as

$$\bar{w}_{2\gamma} = \frac{1}{4} w_o \qquad ...(70)$$

therefore

$$w_o = 4|\psi(0)|^2 \left(v\bar{\sigma}_{2\gamma}\right)_{v \to o}. \qquad ...(71)$$

From equations (68), (69) and (71), we have

$$w_o = \frac{m_o g^{10}}{2\hbar^6 c^3}. \qquad ...(72)$$

Thus the lifetime of non-Abelian para-anti-monopolium is

$$\tau_o = \frac{2\hbar}{m_o c^2 \alpha_g^5} \approx 4 \cdot 65 \times 10^{-63} \text{ seconds.} \qquad ...(73)$$

This process is shown in fig. (1). Similarly, we can show that the decay probability of non-Abelian ortho-anti-monopolium, related to the spin averaged cross section for three-photon decay of a free pair is

$$w_1 = \frac{4}{3}\bar{w}_{3\gamma} = \frac{4}{3}|\psi(0)|^2 \left(v\bar{\sigma}_{3\gamma}\right)_{v \to o}. \qquad ...(74)$$

The statistical weight of a state with spin-1 being 3/4. As such we may mention that

$$\bar{\sigma}_{3\gamma} = \frac{4\left(\pi^2 - 9\right) c\alpha_g g^4}{3 v m_o^2 c^4}. \qquad ...(75)$$

Substituting the value of $\bar{\sigma}_{3\gamma}$ in equation (74) and using equation (69), we get

$$w_1 = \frac{2m_o g^{10}\left(\pi^2 - 9\right)\alpha_g}{9\pi\hbar^6 c^3} \quad . \qquad ...(76)$$

Thus the lifetime for non-Abelian ortho-anti-monopolium is

$$\tau_1 = \frac{9\pi\hbar}{2m_o c^2\left(\pi^2 - 9\right)\alpha_g^6} \approx 1\cdot 24\times 10^{-66} \text{ seconds} \qquad ...(77)$$

which can be represented diagrammatically as shown in fig. (2).

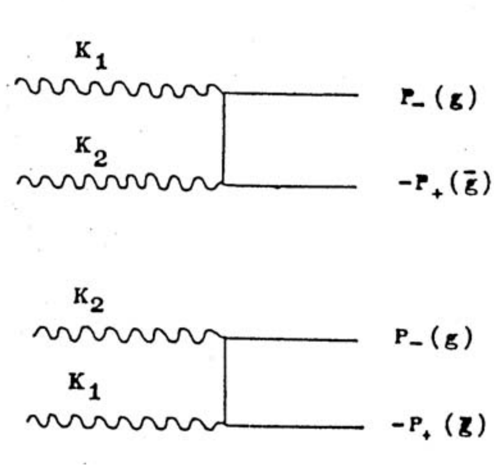

Fig. 1 Two photons decay

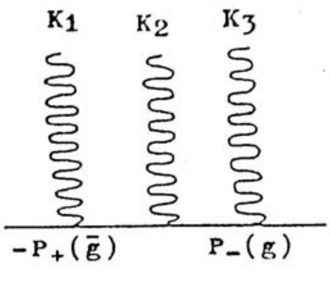

Fig. 2 Three photons decay

In these figures, K1, K2 and K3 are the momenta of photons corresponding to magnetic charge.

5. CONCLUSION

Equation (3) describes the motion of a non-Abelian monopole in the field of another monopole, which on solving gives the extra spin contribution in the energy gained by non-Abelian monopole while moving in the field of another non-Abelian monopole given by equation (13). Equation (16) gives the values of spin moment of non-Abelian monopole which is defined as Bohr magneton for the system. Equation (23) is the relativistic Hamiltonian for non-Abelian monopole in the field of another non-Abelian monopole, different parts of which arise due to different relativistic interactions. Equation (31) is the Lagrangian density for non-Abelian monopole moving in the electromagnetic field of another non-Abelian monopole. Equation (36) is the S-matrix expansion and with the help of Equation (37) we have obtained different scattering processes.

Equation (44) is the scattering amplitude for monopole-anti-monopole system in non-Abelian gauge theory, from which we have obtained the interaction operators $\mathbf{U}$ and $\mathbf{U}^{(ann)}$ described by equations (53) and (61) respectively. Equation (62) is the Hamiltonian for non-Abelian anti-monopolium, equation (67) is the energy levels of non-Abelian para-anti-monopolium and equation (68) is the cross section for two photons annihilation. Equation (73) describes the lifetime of non-Abelian para-anti-monopolium which is 4.65 X 10^{-63} seconds. This lifetime is very small in comparison to the lifetime of para-positronium (1.23 X 10^{-10} seconds).

Equation (77) describes the lifetime of non-Abelian ortho-anti-monopolium (1.24 X 10^{-66} seconds) is also very small in comparison to the lifetime of ortho-positronium (1.4 X 10^{-7} seconds). It implies that this bound state is very small lived and decays in two modes. For non-Abelian para-anti-monopolium, we get almost instantly two photons and for non-Abelian ortho-anti-monopolium, we get three photons that must be extra ordinarily energetic. As such the photon associated with magnetic monopole is certainly different from the photon associated with electron in confirmation with the results of others[(12-17)], where it has been conjectured that exact gauge group for monopoles system is not SU(3) X SU(2) X U(1) but it could be SU(3) X SU(2) X U(1) X U'(1). Moreover, occurrence of monopoles and dyons in heteriotic string theory[(18)] confirms that very high energy is associated with these particles.

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